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**NUMERICAL METHODS FOR THE INVESTIGATION OF THE AERODYNAMIC  
NOISE OF A HELICOPTER ROTOR**

***Abstract.** In this paper, the main numerical methods for modeling the aerodynamically generated rotor noise of helicopter are considered. In particular, finite difference schemes for steady and unsteady 2-dimensional potential flows are given, on the basis of which a model of sound generation of aerodynamic origin of a helicopter rotor blade is built. Examples of the solution of the problem for 3-dimensional potential unsteady flow are considered. It should be noted that until recently there was no general finite difference scheme for solving problems of helicopter rotor acoustics. The numerical-analytical method developed by the author of this paper showed the ability to solve problems of aeroacoustics of a helicopter blade for different types of flow models, both potential and non-potential. The paper gives examples of the solution of similar problems, analyses the application of the numerical-analytical method and compares it with existing finite difference methods. In particular, the calculation schemes for the application of the method for a 2D steady flow and a 3D unsteady flow are presented and the peculiarities of the selection of the number of points in the calculation molecule are explained. Depending on the characteristics of a problem, the number of computational grid points can vary both along the span and across the blade. This makes it possible to adapt the numerical-analytical method for each of the problems to be solved. At the same time, a stable calculation is performed based on the idea of the numerical analytical method itself, and the convergence of the method is performed automatically each time.*

**Keywords:** Helicopter rotor noise, numerical methods, numerical analytical method

**Introduction**

The study of the noise of aerodynamic origin is an important task in the direction of improving modern rotorcraft, in particular helicopters. To successfully solve the problem of noise reduction of aerodynamic origin, it is necessary to find out which of the existing numerical methods are suitable for studying this or that type of flow in which aerodynamic noise is generated. This work is devoted to the analysis of existing numerical schemes for solving problems of noise generation of the aerodynamic origin of a helicopter rotor.

First of all, it should be noted that initially the noise of aerodynamic origin was studied on simple theoretical models, the purpose of which was to establish the dependence of the noise of the rotor rotation on the kinematics of the rotor rotation and the geometry of the blade. But then the range of questions significantly expanded after it was found out that the noise of aerodynamic origin has different components, and types: rotation noise, high-speed impulse noise, as well as the noise of the interaction of vortices and blades.

Mathematical models, equations that simulate various types of noise, differ among themselves. Accordingly, the methods of numerical calculation of rotation noise and blade-vortex interaction noise, for example, are also different. Therefore, we present a detailed analysis of the numerical approaches used to describe various types of the noise of aerodynamic origin.

*The purpose* of this work is to analyze the existing numerical schemes for calculating the noise of the aerodynamic origin of the helicopter rotor and as a result, show the method of helicopter noise simulation depending on the flow regime.

## 1. Analysis of potential flow calculation schemes

The first type of noise that attracted the attention of scientists was the helicopter rotational noise. The first theoretical model that investigated rotational noise was proposed by Gutin [1]. It clearly states that the noise of rotation depends on the generated harmonics and the size of the blade. But this model is only one-dimensional and unfortunately does not reveal the nature of noise as it occurs directly inside the medium.

The next step in the study of the noise of aerodynamic origin is a model of the generation and propagation of small disturbances from a thin wing, which is described by the Kármán-Guderley equation [2], [3], [4]

$$(K - (\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} = 0 \quad (1)$$

where  $K = (1 - M_\infty^2)\delta^{\frac{2}{3}}$  is the transonic similarity parameter,  $\gamma = c_p / c_v$  is the ratio of specific heat capacities. Since this equation is a nonlinear differential equation, the boundary value problems based on it require a numerical solution. Let's consider the existing numerical schemes and perform their analysis

It is known that equation (1), depending on the values of the parameters included in it, allows the simultaneous existence of flow regions with subsonic and supersonic velocities in the flow. This became the reason for selecting the calculation scheme for the numerical solution of the problem based on the equation of (1) it turned out not easy.

The first successful attempt to numerically solve the Kármán -Guderley equation was the Murman-Cole scheme [5]. For the numerical implementation of the scheme, the authors choose equation (1) in a conservative form, which allows using a fully conservative scheme:

$$\left(\frac{w^2}{2}\right)_x - v_{\bar{y}} = 0 \quad (2)$$

where  $w = \phi_x = (\gamma + 1)\phi_x - K$ ,  $v = \phi_{\bar{y}} = (\gamma + 1)\phi_{\bar{y}}$ .

Let  $(x, y)$  is an arbitrary element with a uniform step of the difference grid in the plane  $(x, y)$ , and the nodal points have indices  $(i, j)$ . Equation (2) can be written in a conservative form for a cell that centered at a point  $(i, j)$  (Fig. 1). As a result, we have:

$$\left[\left(\frac{w^2}{2}\right)_{i+1/2,j} - \left(\frac{w^2}{2}\right)_{i-1/2,j}\right]\Delta\bar{y} - (v_{i,j+1/2} - v_{i,j-1/2})\Delta x = 0 \quad (3)$$

Otherwise, relation (3) can be written as:

$$\frac{1}{2}(w_{i+1/2,j} - w_{i-1/2,j})(w_{i+1/2,j} + w_{i-1/2,j})\Delta\bar{y} - (v_{i,j+1/2} - v_{i,j-1/2})\Delta x = 0 \quad (4)$$

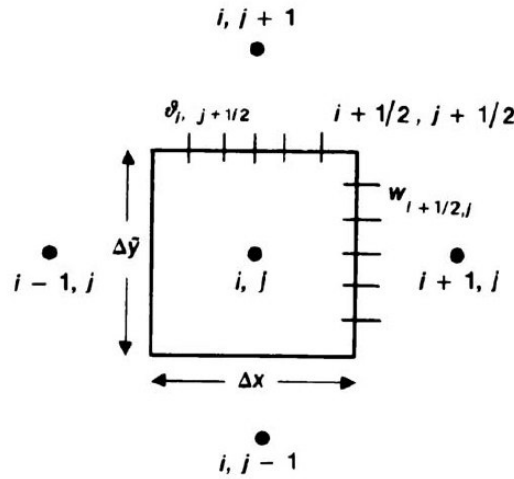


Fig.1 Control cell

Now, if we solve the equation in a domain where it is an equation of elliptic type, that is, an equation of the Laplace type, then central finite differences are used to approximate both  $w$  and  $v$ :

$$\begin{aligned}
 v_{i,j+1/2} &\equiv (\varphi_{\bar{y}})_{i,j+1/2} = \frac{\varphi_{i,j+1} - \varphi_{i,j}}{\Delta \bar{y}}, v_{i,j-1/2} = \frac{\varphi_{i,j} - \varphi_{i,j-1}}{\Delta \bar{y}} \\
 w_{i,j+1/2}^c &\equiv (\varphi_x)_{i+1/2,j} = \frac{\varphi_{i+1,j} - \varphi_{i,j}}{\Delta \bar{y}}, w_{i,j-1/2} = \frac{\varphi_{i,j} - \varphi_{i,j-1}}{\Delta \bar{y}} \\
 \left(\frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta x}\right) \left(\frac{\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i-1,j}}{(\Delta x)^2}\right) - \left(\frac{\varphi_{i,j+1} - 2\varphi_{i,j} + \varphi_{i,j-1}}{(\Delta \bar{y})^2}\right) &= 0 \tag{5}
 \end{aligned}$$

Equation (5) is a central difference scheme of the second order of accuracy, which is an approximation of the equation for the elliptical (subsonic) region. The issue of the stability of this scheme and fulfillment of the Courant-Friedrichs-Levy criterion is discussed in detail in the monograph of one of the authors of this scheme [4]:

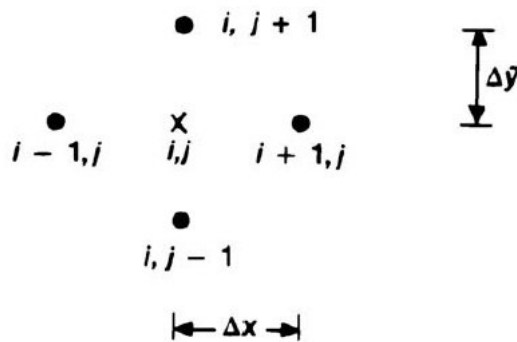


Fig. 2. Calculation template in the elliptic domain

Such a template is not suitable for the hyperbolic (sonic) flow region, since it contains points upstream. To avoid this effect, the calculation template shifts downstream by one point by index  $i + 1 \rightarrow i$  for the first (nonlinear) term of equation (2), which  $i$  is responsible for the formation of shock waves that transform into a sound wave:

$$\left(\frac{\varphi_{i,j} - \varphi_{i-2,j}}{2\Delta x}\right)\left(\frac{\varphi_{i,j} - 2\varphi_{i-1,j} + \varphi_{i-2,j}}{(\Delta x)^2}\right) - \left(\frac{\varphi_{i,j+1} - 2\varphi_{i,j} + \varphi_{i,j-1}}{(\Delta y)^2}\right) = 0 \quad (6)$$

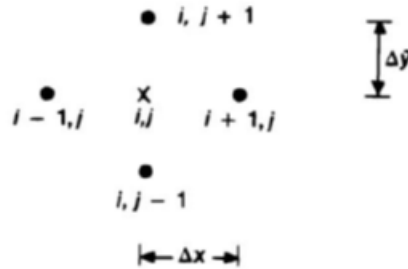


Fig.3 Calculation template in the hyperbolic domain

What happens in the region of sound points: points where the flow accelerates or decelerates to the speed of sound? A different pattern is used here, which does not coincide with either the hyperbolic or elliptical patterns:

$$\left(\frac{\varphi_{i+1,j} - \varphi_{i,j} + \varphi_{i-1,j} + \varphi_{i-2,j}}{2\Delta x}\right)\left(\frac{\varphi_{i+1,j} - \varphi_{i,j} + \varphi_{i-1,j} + \varphi_{i-2,j}}{(\Delta x)^2}\right) - \left(\frac{\varphi_{i,j+1} - 2\varphi_{i,j} + \varphi_{i,j-1}}{(\Delta y)^2}\right) = 0 \quad (7)$$

As can be seen from the above example of solving the problem, it is impossible to use only one numerical template for different flow regions. The finite-difference scheme considered above is a scheme of the first order of accuracy in the hyperbolic flow region [5]. In the elliptical flow region, it has the second order of accuracy.

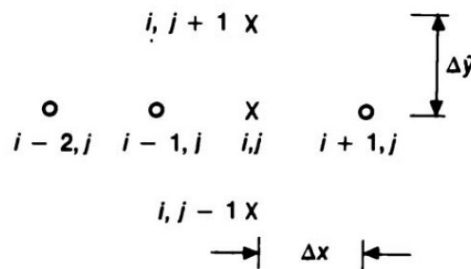


Fig. 4 Calculation template of the operator at a point on the shock wave

The second-order accuracy difference schemes for the hyperbolic domain are given in [5], and the methods for obtaining them are described in the monograph [6]. Analysis of coupled shock waves, in terms of using numerical schemes, was carried out in [7]. After it, there have already appeared works using this calculation procedure [8], but for equations in full potentials, without separation of small perturbations from the flow. In [8] calculation of plane unsteady flow has been carried out. Both central and mixed finite differences were used for mixed time and coordinate derivatives.

So far, we have not talked about the characteristics of the flow during study we are interested in shock waves. For the analysis of the solution of the Kármán -Guderley equation, which describes the propagation of small disturbances from a thin wing, the pressure coefficient  $C_p$  is of interest. The pressure coefficient is defined as a ratio  $C_p = 2(p - p_\infty) / \rho_\infty U^2$ . The above numerical schemes calculate the pressure coefficient in the form of a parabola if there is no shock wave in the flow. If a gap appears on the pressure curve, that is, a shock wave is realized, then the profile has the form of a deformed, "torn" parabola [4]:

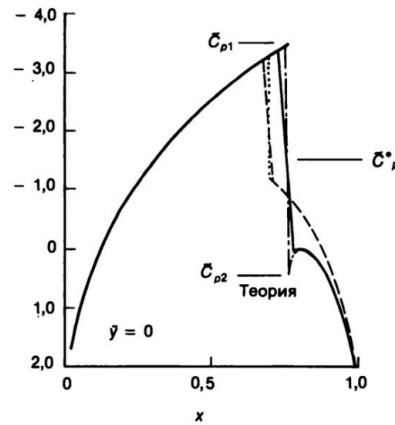


Fig. 5 Pressure distribution,  $K = 1.8$

The experience of later studies of the unsteady plane transonic flow showed that the pressure coefficient of propagation of small disturbances in the unsteady flow has a different shape than a parabola. Such a discrepancy can be explained by the fact that the numerical schemes considered in reality do not reveal the non-stationarity of the flow: in them, the shock wave is, as it were, stopped in relation to time. Therefore, it makes sense to further consider non-stationary schemes for solving the Kármán-Guderley equation.

In the second half of the 70s of the last century, to solve a two-dimensional non-stationary problem for the equation of propagation of small disturbances from thin wings, a number of authors resorted to an implicit ADI (alternating direction implicit). This method was used in [9] for the case of low-frequency unsteady two-dimensional flows. In this case, the Kármán-Guderley equation is solved in the following form:

$$A\phi_{tt} + 2B\phi_{xt} = C\phi_{xx} + \phi_{yy} \quad (8)$$

where

$$A = k^2 M_\infty^2 / \delta^{2/3}, B = k M_\infty^2 / \delta^{2/3}, C = (1 - M_\infty^2) / \delta^{2/3} - (\gamma + 1) M_\infty^m \phi_x$$

The parameter  $m$  of this expression is a function of  $M_\infty$ . This parameter is an adjustment parameter for the critical value of the pressure coefficient [10]. The time parameter  $k = \omega c / U_\infty$  is the Strouhal number. According to the coordinates  $x, y$  in (8), the double-sweep method was used:

by coordinate  $x$ :

$$2B(\Delta t)^{-1} \delta_x (\bar{\phi}_{j,k}^{n+1} - \phi_{j,k}^n) = D_x f_{j,k} + \delta_{yy} \phi_{j,k}^n$$

by coordinate  $y$ :

$$2B(\Delta t)^{-1} \delta_x(\phi_{j,k}^{n+1} - \bar{\phi}_{j,k}^n) = \frac{1}{2} \delta_{yy}(\phi_{j,k}^{n+1} - \phi_{j,k}^n), B = kM_\infty^2 / \delta^{2/3},$$

$$\delta_x \phi_{j,k} = 2(\phi_{j,k} - \phi_{j-1,k})(x_{j+1} - x_{j-1})^{-1} = (3\phi_{j,k} - 4\phi_{j-1,k} + \phi_{j-2,k})(x_{j+1} - x_{j-1})^{-1},$$

$$\delta_x \phi_{j,k} = 2(\phi_{j,k} - \phi_{j-1,k})(x_{j+1} - x_{j-1})^{-1} = (3\phi_{j,k} - 4\phi_{j-1,k} + \phi_{j-2,k})(x_{j+1} - x_{j-1})^{-1},$$

$$\delta_{yy} \phi_{j,k} = 2[(\phi_{j,k+1} - \phi_{j,k})(y_{k+1} - y_{k-1})^{-1} - (\phi_{j,k} - \phi_{j,k-1})(y_k - y_{k-1})^{-1}](y_{k+1} - y_{k-1})^{-1}. \quad (9).$$

$$f_{j,k} = \frac{1}{2} [C_{j,k}^n \phi_{x_j,k}^{n+1} + (1 - M_\infty^2) \phi_{x_j,k}^n / \delta^{2/3}],$$

$$\phi_{x_{j+1/2},k} = (\phi_{x_{j+1},k} - \phi_{x_j,k})(x_{j+1} - x_{j-1}),$$

$$Df_{j,k} = 2(x_{j+1} - x_{j-1})^{-1} [(1 - \varepsilon_j)(f_{j+1/2,k} - f_{j-1/2,k}) + \varepsilon_{j-1}(f_{j-1/2,k} - f_{j-3/2,k})] \quad (10)$$

Here

$$\varepsilon_j = 0, \text{ if } C_{j+1/2,k}^n + C_{j-1/2,k}^n > 0, \varepsilon_j = 1, \text{ if } C_{j+1/2,k}^n + C_{j-1/2,k}^n < 0.$$

This work was followed by more publications in which the method of modeling the flow at the shock wave is being improved. Thus, in works [61], [54], the method of separation of jumps and stretching of grid coordinates is used. Shock waves are considered as gaps perpendicular to the flow direction. A little later [13] the authors develop their approach for three-dimensional flow, but not for Kármán-Guderley, and for the equation of the full flow potential. At the beginning of the 80s [14] the inverse problem is solved: according to the given pressure is the flow field and the shape of the wing profile.

## 2. Numerical analytical method

The author of this work proposed a numerical-analytical method [15], [16], which has been successfully tested on a number of solved problems of helicopter blade noise generation. The main idea of the method is that finite-difference representations of derivatives in the equations to be solved are not used explicitly. The reason is simple: it is not always known in advance which of the known decompositions, with what accuracy, will prove to be the most convenient.

Therefore, according to the idea of the method, we have the following: at the  $n-1$  points of the calculated "molecule" we perform a standard Taylor series expansion and at the  $n$  point we assume that the small perturbation propagation equation is automatically fulfilled. In fact, we forcibly require that the small unsteady perturbation generation equation is satisfied at the  $n$ -th point. From such condition we actually have automatic fulfillment of convergence condition of required numerical solution.

At the same time, the expansion coefficients in the Taylor series are expressed implicitly from the system of  $n$  equations. Such a scheme of the method is used for separate boundary value problems, where the boundary condition allows integration, therefore it is not added to the calculation system of equations. But this happens only in simple cases.

If it is additionally necessary to fulfill a boundary condition, which occurs in most problems, then this boundary condition is also added to the calculation system of equations. In this case, we have the following: at the  $n-2$  points of the calculated "molecule" we perform

expansion into a multidimensional Taylor series, and we also require satisfying of the equation being solved and the boundary condition. Thus, the number of points of the calculated molecule, for which expansion in the Taylor series is required, is reduced by 2.

Below we will consider the implementation of the numerical analytical method for the two-dimensional stationary and three-dimensional non-stationary Kármán-Guderley equations.

### 3. 2-dimensionals case

Equation (1) of Kármán-Guderley in dimensionless variables  $\xi = x/c, \eta = \lambda y$  looks like :

$$\left[1 - \frac{1}{M_1^2} + \varepsilon \cdot (\gamma + 1) f_\xi\right] f_{\xi\xi} - \frac{\lambda^2}{M_1^2} f_{\eta\eta} = 0, \quad (11)$$

For the two-dimensional case [17], we have a 6-point scheme: at five points we expand the function  $f(\xi_i, \eta_i), f \in C^2([0;1] \times [0;1])$ , into a Taylor series:

$$\begin{aligned} f(\xi_i, \eta_i) &= f(\xi_0, \eta_0) + f_\xi(\xi_0, \eta_0)(\xi_i - \xi_0) + f_\eta(\xi_0, \eta_0)(\eta_i - \eta_0) + \\ &+ \frac{1}{2} [f_{\xi\xi}(\xi_0, \eta_0)(\xi_i - \xi_0)^2 + 2f_{\xi\eta}(\xi_0, \eta_0)(\xi_i - \xi_0)(\eta_i - \eta_0) + f_{\eta\eta}(\xi_0, \eta_0)(\eta_i - \eta_0)^2] + \\ &+ o(\max\{(\xi_i - \xi_0)^2, (\xi_i - \xi_0)(\eta_i - \eta_0), (\eta_i - \eta_0)^2\}), \quad i = 1, \dots, 5. \end{aligned}$$

In the 6th point  $(\xi_0, \eta_0)$ , we require automatic execution of the equation:

$$\left[1 - \frac{1}{M^2} + \varepsilon(\gamma + 1) f_\xi(\xi_0, \eta_0)\right] f_{\xi\xi}(\xi_0, \eta_0) - \frac{\lambda^2}{M_1^2} f_{\eta\eta}(\xi_0, \eta_0) = 0. \quad (13)$$

The non-penetrating boundary condition  $f_\eta = \delta g_\xi, 0 < \xi < 1$  in this problem is fulfilled automatically.

This numerical scheme allowed direct satisfaction of the convergence of the proposed method, and its implicit type ensured the stability of the calculation. The curves of pressure coefficients given in Fig.6 at Mach number values  $M \approx 1$  (the range of the problem calculation zone) indicate that the numerical analytical method successfully coped with the task of calculating the sound flow in a physically unstable region. At the same time, the numerical scheme did not "fall apart", as happened with most known finite-difference representations.

Note that in the case where function  $f(\xi_i, \eta_i, \tau_i)$  depends on time, 4 more derivatives  $f_\tau, f_{\tau\tau}, f_{\xi\tau}, f_{\eta\tau}$  are added to the Taylor series in the expansion, so the scheme becomes an 8-points scheme if we take into account the fulfillment of the boundary condition.

### 4. 3-dimensionals case

The author of this paper previously derived a complete three-dimensional equation for the propagation of small disturbances from a thin wing [18], and performed an analysis of its

partial cases. To solve the boundary problem of studying the generation and propagation of small non-stationary disturbances from a helicopter blade, a numerical analytical method was used [19]. In this case of its application, we have a 3-dimensional equation for the propagation of small disturbances:

$$\left(\frac{kc}{U}\right)^2 f_{\tau\tau} + \left[1 - \frac{1}{M_1^2} + (1 + \gamma)\varepsilon f_{\xi}\right] f_{\xi\xi} + \frac{2kc}{U} f_{\xi\tau} - \frac{(\lambda c)^2}{M_1^2} f_{\eta\eta} - \left(\frac{c}{R}\right)^2 \frac{1}{M_1^2} f_{\zeta\zeta} = 0 \quad (14)$$

where  $\xi = x/c, \eta = \lambda y, \zeta = z/R, \tau = kt$ .

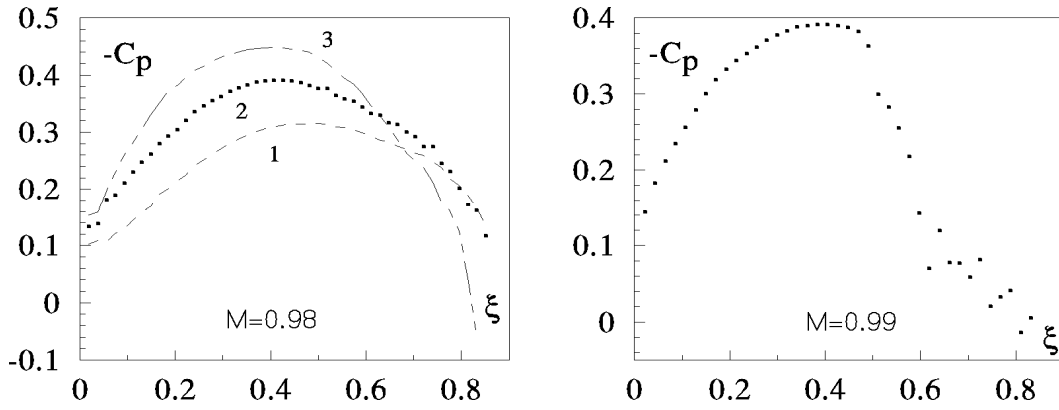


Fig. 6 Calculation of the pressure coefficient in the transonic flow range

Boundary condition is:

$$\delta \left[ \frac{kc}{U} g_{\tau} + g_{\xi} \right] = \varepsilon \lambda c f_{\eta}, 0 < \xi < 1, \eta = \eta(\xi), \quad (15)$$

Next, by the analogy given above for the two-dimensional case, we choose the number of calculation points of the template by the number of unknown derivatives in the function represented in the Taylor series expansion, but by 2 less, since we also have the boundary condition (14) and equation (15), which is solved.

$$\begin{aligned} f(\xi_i, \eta_i, \zeta_i, \tau_i) &= f(\xi_o, \eta_o, \zeta_o, \tau_o) + f_{\xi}(\xi_i - \xi_o) + f_{\eta}(\eta_i - \eta_o) + f_{\zeta}(\zeta_i - \zeta_o) + f_{\tau}(\tau_i - \tau_o) + \\ &+ \frac{1}{2} [f_{\xi\xi}(\xi_i - \xi_o)^2 + f_{\eta\eta}(\eta_i - \eta_o)^2 + f_{\zeta\zeta}(\zeta_i - \zeta_o)^2 + f_{\tau\tau}(\tau_i - \tau_o)^2] + f_{\xi\eta}(\xi_i - \xi_o)(\eta_i - \eta_o) + \\ &+ f_{\xi\zeta}(\xi_i - \xi_o)(\zeta_i - \zeta_o) + f_{\eta\zeta}(\zeta_i - \zeta_o)(\eta_i - \eta_o) + f_{\xi\tau}(\xi_i - \xi_o)(\tau_i - \tau_o) + f_{\eta\tau}(\eta_i - \eta_o)(\tau_i - \tau_o) + \\ &+ f_{\zeta\tau}(\zeta_i - \zeta_o)(\tau_i - \tau_o) + R(\Delta^3), i = 1, n-2 \end{aligned} \quad (16)$$

So, in the three-dimensional non-stationary case, we have shown the calculation of the pressure coefficient (Fig.7) in the cross-section of the blade. As can be seen from Fig.7, the pressure distribution according to the non-stationary model is significantly different from the pressure distribution according to the stationary model (Fig.6).

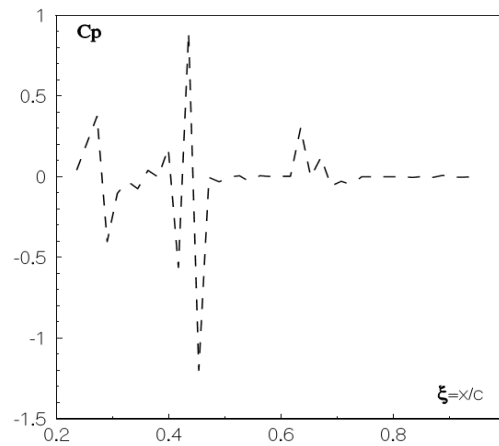


Fig. 7 Pressure coefficient along the blade chord

### 5. Calculation schemes of non-potential flow

The existing schemes for calculating sound disturbances in the potential approximation were considered above. These models are usually used for relatively large values of Mach numbers,  $M > 0,5$ , when rotational noise is dominant, rather than vortex noise. In the case of  $M < 0,5$ , the vortex component of the noise is significantly activated, and therefore the potential approximation is no longer appropriate. In this case, full equations for simulating the generation of sound of aerodynamic origin, which take into account the vortex component of the flow, are used for calculation.

To date, a sufficient number of theoretical non-potential models [20], [21] are known which, according to the authors, are capable of describing the generation of the sound of aerodynamic origin. However, these models are somewhat physically inaccurate. That is why the majority of researchers choose the currently most popular models of Fowkes-Williams-Hawkings [22] and Forsyth's 1A formulation [23] for the calculation of helicopter noise, and Lighthill's acoustic analogy is used only in some cases. But this does not mean that these models are physically more accurate: Lighthill's acoustic analogy and its application in the form of the Ffowkes-Williams-Hawkings model leave the question of the physicality of these approaches [21].

Consider the Ffowkes-Williams-Hawkings equation:

$$4\pi^2(\rho - \rho_0) = \frac{\partial^2}{\partial x_i \partial x_j} \int_{V_0} \left[ \frac{T_{ij}}{r|1-M_r|} \right] d\eta - \frac{\partial}{\partial x_i} \int_S \left[ \frac{P_{ij} n_j}{r|1-M_r|} \right] dS(\eta) - \frac{\partial^2}{\partial x_i \partial x_j} \int_{V_0} \left[ \frac{\rho_0 v_i v_j}{r|1-M_r|} \right] d\eta + \frac{\partial^2}{\partial x_i \partial x_j} \int_{V_0} \left[ \frac{\rho_0 v_i v_j}{r|1-M_r|} \right] d\eta \quad (17)$$

Equation (17) contains the Lighthill equation inside, which was obtained artificially: by adding a certain term to the left and right sides, which formally led it externally to a wave equation that seems to describe the generation of sound. The question arises: if we add other derivatives to the left and right sides, what will we get formally? I mean that the approach used by Lighthill is not physically correct, because by physically unexplained addition and subtraction of certain derivatives in the equation, we thereby actually add certain physical

components, and get equations that describe a completely different physical process. This is the main physical drawback of this approach.

### Formulation 1 of Farassat:

According to the formula obtained by Farassat [23], the sound pressure is determined through the thickness source  $p'_T(x, t)$  and the load source  $p'_L(x, t)$ :

$$4\pi p'(x, t) = 4\pi(p'_T(x, t) + p'_L(x, t)) = \frac{\partial}{\partial t} \int_{f=0} \left[ \frac{\rho_0 v_n}{r(1-M_r)} + \frac{p \cos \theta}{cr(1-M_r)} \right]_{ret} dS + \int_{f=0} \left[ \frac{p \cos \theta}{r^2(1-M_r)} \right]_{ret} dS \quad (18)$$

where  $\cos \theta = n_i \hat{r}_i$  is the local angle between the normal to the surface and the radiation direction.

If you look carefully at equations (17) and (18), you can make sure that nothing is said about the definition of the near sound field in these approaches. What methods are used to find the potential of the sound field, or the density in the sound wave - they do not write about it.

The main disadvantage of using Farassat's approach is that the source of sound is non-existent fictitious sources, which are placed inside a solid surface. The question arises: why is such a strange approach used? The answer is simple: in classical acoustics, Green's function is used as an auxiliary solution to represent the far sound field. This makes it possible to record the specified representation of the far field based on second Green's formula for real sound sources.

Farassat used the next approach: he said that these sources do not have to be real sound sources, let them be imaginary ones. Moreover, they are placed inside a hard surface that does not emit sound. And this is actually the main obstacle to a physically accurate determination of aerodynamically generated sound: in fact, such an approach lacks real sound sources. And this was discussed in the work of Fedorchenko [24], as well as in the work [18].

In addition to the above-mentioned models, there are several models that describe the sound of aerodynamic origin [21], but there is no numerical implementation of them. An exception is a theoretical model proposed by the author of this paper [21]. The closed system of equations obtained based on this model makes it possible to describe the problems of aeroacoustics with accuracy to the values of the second order of smallness:

$$\frac{\partial^2 \bar{\rho}'}{\partial \tau^2} - \frac{1}{M_\infty^2} \cdot \frac{\partial^2 \bar{\rho}'}{\partial \xi^2} - a^2 (\lambda^2 c^2 \cdot \frac{\partial^2 \bar{\rho}'}{\partial \eta^2} + \frac{1}{AR^2} \cdot \frac{\partial^2 \bar{\rho}'}{\partial \zeta^2}) + R(\bar{\rho}', \frac{\partial \bar{\rho}'}{\partial \xi}, \frac{\partial \bar{\rho}'}{\partial \eta}, \frac{\partial \bar{\rho}'}{\partial \zeta}, \frac{\partial^2 \bar{\rho}'}{\partial \xi^2}, \frac{\partial^2 \bar{\rho}'}{\partial \eta \partial \xi}, \dots, \frac{\partial^2 \bar{\rho}'}{\partial \zeta^2}) =$$

$$\gamma \left( \frac{\partial \bar{\varphi}}{\partial \xi}, \frac{\partial \bar{\varphi}}{\partial \eta}, \frac{\partial \bar{\varphi}}{\partial \zeta}, \frac{\partial^2 \bar{\varphi}}{\partial \xi^2}, \frac{\partial^2 \bar{\varphi}}{\partial \eta \partial \xi}, \dots, \frac{\partial^3 \bar{\varphi}}{\partial \zeta^3} \right), \quad (19)$$

$$\bar{\rho} \left( \frac{\partial^2 \bar{\varphi}}{\partial \xi^2} + c^2 \lambda^2 \frac{\partial^2 \bar{\varphi}}{\partial \eta^2} + \frac{1}{AR^2} \frac{\partial^2 \bar{\varphi}}{\partial \zeta^2} \right) + c \frac{\partial \bar{\rho}}{\partial \xi} \cdot \frac{\partial \bar{\varphi}}{\partial \xi} + c^2 \lambda^2 \frac{\partial \bar{\rho}}{\partial \eta} \cdot \frac{\partial \bar{\varphi}}{\partial \eta} + \frac{1}{AR^2} \frac{\partial \bar{\rho}}{\partial \zeta} \cdot \frac{\partial \bar{\varphi}}{\partial \zeta} = - \left[ c \frac{\partial \rho'}{\partial \tau} +$$

$$\bar{\rho}' \left( c \frac{\partial \bar{u}}{\partial \xi} + \lambda c^2 \frac{\partial \bar{v}}{\partial \eta} + \frac{c^2}{R} \frac{\partial \bar{w}}{\partial \zeta} \right) + c \bar{u} \frac{\partial \bar{\rho}'}{\partial \xi} + \lambda c^2 \bar{v} \frac{\partial \bar{\rho}'}{\partial \eta} + \frac{c^2}{R} \bar{w} \frac{\partial \bar{\rho}'}{\partial \zeta} \right] \quad (20)$$

In equations (19)-(20),  $\bar{\varphi}, \bar{\rho}'$  - are acoustical density and sound potential correspondingly.

Despite the mathematical complexity of this system of equations, the author managed to numerically solve a number of interesting problems using the numerical analytical method [25]-[28]. An interesting feature of the application of the numerical analytical method, which was noticed, is the use of an unstructured calculation grid around the blade. So, for example, along the chord, the step of the calculated grid is smaller (Fig. 8, Fig. 9) than along the span of the blade. This is necessary for stable calculation according to the numerical analytical method.

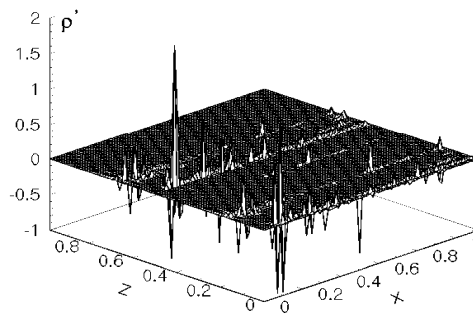


Fig.8 Distribution of acoustic density on the surface of a helicopter blade [25]

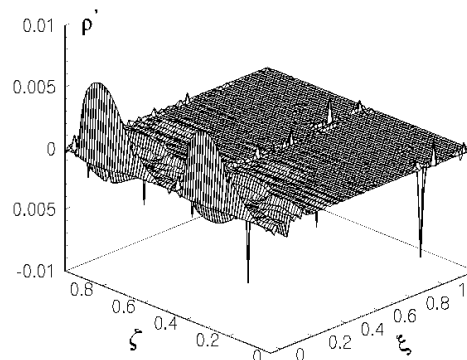


Fig. 9 Distribution of acoustic density on the surface of a helicopter blade [26]

And in conclusion, we will indicate that the near-field calculation data obtained with the application numerical-analytical approach is easy to use to calculate the integral representation of the far field. For this, there is no need to use any specific numerical approaches: standard methods of calculating definite integrals are used.

## Conclusions

1. In this work, a study of the methods of numerical calculation of the near sound field of a helicopter is carried out.
2. The conducted analysis showed that there was still no single method, a finite-difference scheme, for various flow models: each of the given numerical schemes solved a certain problem.
3. It is shown that the numerical analytical method is capable of solving the problems of helicopter noise generation for various types of flow, both potential and non-potential. This

feature of the method allows it to be used for various tasks of modeling noise of aerodynamic origin.

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## **ЧИСЕЛЬНІ СХЕМИ ДОСЛІДЖЕННЯ АЕРОДИНАМІЧНОГО ШУМУ РОТОРА ГЕЛІКОПТЕРА**

***Анотація.** В даній роботі розглянуто основні чисельні схеми моделювання шуму ротора гелікоптера аеродинамічного походження. Зокрема, наведено кінцево-різницеві схеми для стаціонарної, нестаціонарної 2-вимірної потенціальної течії, на основі якої будується модель генерації звуку аеродинамічного походження лопаті ротора гелікоптера. Наведено приклади розв'язання задачі для 3-вимірної потенціальної нестаціонарної течії. Для кожної з перелічених задач пропонується власна кінцево-різницева схема. Ці схеми суттєво відрізняються одна від одної. Отже єдиної кінцево-різницевої схеми для розв'язання задач акустики ротора гелікоптера фактично не існувало до недавніх пір. Це ускладнювало практичне застосування вказаних схем, оскільки під час розв'язання конкретної задачі можлива реалізація різних типів течії, що потребує постійної зміни розрахункового шаблону. Вирішення даної проблеми вдалось досягнути завдяки використанню чисельно-аналітичного методу. Чисельно-аналітичний метод, розроблений автором даної роботи, показав спроможність розв'язувати задачі аероакустики лопаті гелікоптера для різних типів течії, як потенціальних так і не потенціальних. В роботі наводяться приклади розв'язання подібних задач, виконано аналіз застосування чисельно-аналітичного методу, порівняння з існуючими кінцево-різницевиими схемами. Зокрема, представлені розрахункові шаблони застосування методу для 2-вимірної стаціонарної течії, 3-вимірної нестаціонарної течії, пояснена специфіка вибору кількості точок у розрахунковій молекулі. В залежності від особливості конкретної задачі, кількість точок розрахункової сітки може варіюватись як уздовж розмаху, так і поперек лопаті. Це дає змогу налаштовувати чисельно-аналітичний метод під кожну з задач, що розв'язується. При цьому виконується стійкий розрахунок, а збіжність методу щоразу виконується автоматично на підставі ідеї самого чисельно-аналітичного методу. Розроблений автором метод дозволяє виконувати чисельний розрахунок шуму ротора гелікоптера для різних режимів його роботи.*

***Мета статті.** Метою даної роботи є аналіз існуючих чисельних схем розрахунку шуму аеродинамічного походження ротора гелікоптера.*

***Методи дослідження.** Теоретичний аналіз існуючих чисельних схем.*

***Результати дослідження.** Виявлено відсутність єдиного чисельного підходу до розв'язання задач генерації шуму аеродинамічного походження ротора гелікоптера. У якості розв'язання даної проблеми пропонується чисельно-аналітичний метод, розроблений автором даної роботи.*

***Висновки.***

*У даній роботі виконано дослідження методів чисельного розрахунку ближнього звукового поля гелікоптера, котрі використовуються для різних фізичних моделей: потенціальна течія та не потенціальна течія.*

*Проведений аналіз показав, що єдиного методу, кінцево-різницевої схеми, для різних моделей течії до сих пір не було: кожна з наведених чисельних схем розв'язувала лише певну задачу.*

*Показано, що чисельно-аналітичний метод здатен розв'язувати задачі генерації шуму гелікоптера для різних типів течії, як потенціальної, так і не потенціальної. Ця особливість методу дозволяє використовувати його для різних задач з моделювання шуму аеродинамічного походження.*

**Ключові слова:** шум ротора гелікоптера, чисельні методи, чисельно-аналітичний метод.

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## **РОЗРОБКА ВЕБ-ОРІЄНТОВАНОГО ІНСТРУМЕНТАЛЬНОГО СЕРЕДОВИЩА ДЛЯ КЕРУВАННЯ РОЗКЛАДОМ ЗАНЯТЬ**

*У даній статті представлено розробку веб-орієнтованого інструментального середовища для ефективного керування розкладом занять в навчальних закладах. Зростання кількості учнів та викладачів у сучасних освітніх установах ставить перед керівництвом виклик в ефективному плануванні та уникненні конфліктів у графіку навчальних занять. Програмний інструмент, що пропонується, надає зручний інтерфейс для складання та оновлення розкладу, дозволяючи враховувати різноманітні обмеження, наявність різних груп студентів, викладачів та аудиторій. Призначений інструмент допомагає автоматизувати процес планування, забезпечує оптимальне розподілення ресурсів та допомагає уникнути помилок, що можуть виникати при складанні розкладу вручну. Результати експериментів підтверджують ефективність запропонованого інструменту та переваги використання веб-технологій для керування розкладом навчальних занять.*

**Ключові слова:** розклад, автоматична генерація розкладу, генетичний алгоритм, веб-додаток.