СЕКЦІЯ «ПРИКЛАДНА МАТЕМАТИКА»

UDK 532.517.4

PACS 02.60.-x, 02.60.Pn, 02.70.Wz

GOLOVNYA Boris P.
Department of Informatics and Applied Mathematics, The Bohdan Khmelnytsky National University of Cherkasy, Ukraine
e-mail: BPGolovnya@gmail.com

ON CASCADE ENERGY TRANSFER IN TURBULENCE MODELING

Cascade energy transfer plays a very important role in all turbulent flows. Unfortunately, this process is very difficult to model. This paper analyzes one of the possible causes of difficulties. It is shown that the turbulence model proposed earlier by the author successfully copes with these difficulties.

Key words: *cascade transfer modeling, k-ε turbulence model.*

1. The importance of cascade transfer

Almost all modern models of turbulence are based on the equation of turbulent energy transfer. If such model is reckoned as universal then it must answers three questions. First – where does the turbulent energy originate? Second – what happens to this energy? Third – how does the energy dissipate? Otherwise there are certainly exists such flow that this model can not reproduce.

For the vast majority of turbulent flows Kolmogorov's theory of cascade energy transfer answers these questions. By this reason results of simulations by high-quality models must be in agreement with this theory.

In this work, we will try to figure out which properties allow some models to simulate cascade energy transfer.

2. The Jones-Launder model

The Jones-Launder model [1] has the following form

$$\begin{cases} \dfrac{Dk}{Dt} = \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_t}{C_k}\right)\dfrac{\partial k}{\partial x_k} + P - \varepsilon, \\ \dfrac{D\varepsilon}{Dt} = \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_t}{C_\varepsilon}\right)\dfrac{\partial \varepsilon}{\partial x_k} + \dfrac{\varepsilon}{k}\left(C_1 P - C_2 \varepsilon\right). \end{cases} \quad (1)$$

It is stated in [2] that the equations system

$$\begin{cases} \dfrac{dk}{dt} = -\varepsilon, \\ \dfrac{d\varepsilon_e}{dt} = -C_2\dfrac{\varepsilon^2}{k}. \end{cases} \quad (2)$$

reproduces well the grid turbulence decay if C_2 = 1.4. Note that it was shown in [3] that the requirement C_2 <1.5 in (2) follows from the condition $\lim_{t\to\infty} L_e = 0$. Based on the coincidence of the dissipative terms of systems (1) and (2), we assume that system (2) is a simplified Jones-Launder model (1). Because the grid turbulence decay is very similar to a cascade process, it was decided to test this model for the possibility of the cascade process simulating.

The model for the cascade process simulation is based on the following hypothesis. It is supposed that the dissipative term describes not the rate of the transition of the kinetic energy to the thermal energy, but the rate of energy transfer into the cascade process. On the base of this supposition we calculated several steps of a discrete cascade process. Each step of the process was calculated on a separate model. Therefore, the complete model of process consists of several similar systems of equations. The turbulence model as such describes the first step of the cascade process. Its dissipative term describes the transfer of energy to the second step of the cascade. Therefore, it is used as the production term in the second step. The dissipation of the second step is used as the production in the third step, and so on.

One must say that a similar model, although for other purposes, was proposed in the [4]. But any real using of this model was not found in the literature.

3. Governing equations

At the first step of the calculation, system (1) is solved. The equations for modeling of the next step of a discrete cascade process looks as follows. Here i is the number of the cascade process step.

$$\begin{cases} P^i = \varepsilon^{i-1} \\ \dfrac{Dk^i}{Dt} = \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_t^i}{C_k}\right)\dfrac{\partial k^i}{\partial x_k} + P^i - \varepsilon^i, \\ \dfrac{D\varepsilon^i}{Dt} = \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_t^i}{C_\varepsilon}\right)\dfrac{\partial \varepsilon^i}{\partial x_k} + \dfrac{\varepsilon^i}{k^i}\left(C_1 P^i - C_2 \varepsilon^i\right). \end{cases} \quad (3)$$

The initial conditions of the whole simulation and are the values of energy and dissipation k^0 and ε^0 obtained from solution (1).

The initial conditions for calculating the next step of the cascade process are set as

$$k_0^j = \alpha k_0^{j-1},\ \varepsilon_0^j = \varepsilon_0^{j-1} = \varepsilon_0^0 . \quad (4)$$

4. Solution requirements

1. The turbulent energy spectrum must satisfy the relation $E(\kappa) \sim \kappa^{-5/3}$. Here E is the energy function, κ is the wave number. In this paper, the values of the discrete energy function were found as $E_i(\kappa) = k_i/\kappa_i$, the values of the discrete wave number - $\kappa_i = 2\pi/L_i$. Here k_i is the turbulent energy of the i-th step of the cascade process, L_i is the value of the dissipative scale at this step.
2. The transition of the energy of turbulence to heat occurs at small scales of the vortices. It follows that the energy transmitted through the cascade per time unit, i.e. the rate of dissipation, is unchanged.
3. The values of turbulent energy in any cross section of a turbulent vortex decrease during the cascade process. Reason. In the cascade process vortices stretch. So their radiuses decrease. Decrease of vortices radiuses leads to decrease of linear velocities of vortices. As a result the levels of fluctuations decrease also.

4. The diameters of the vortices decrease in the process of cascade transfer. Hence, if we assume that the dissipative scale is a characteristic of the diameter of the vortices, then it must also decrease.

The following must be said. These are fairly general requirements for modeling a cascade process. But, if the model poorly reproduces the turbulence energy profile, turbulent viscosity, etc., then it is impossible to require that the solutions of the equations of the model satisfy all these requirements. Therefore, here we check that the results match only the first item on the list. In this regard, we mention that (see, for example [5]) the flow in round jet cannot be predicted with the standard k-e and therefore with Jones-Launder model.

5. Results of simulations

It is known that the Jones-Launder model poorly calculates near-wall turbulence. Therefore, we will check the capabilities of the model by calculating a cascade process in free flows - a round jet and a flat shear layer. In these cases, both the turbulence itself and the cascade process were simulated using the Jones-Launder model.

Figures 1 and 2 show the results of modeling the cascade transfer in a round jet and a plane shear layer.

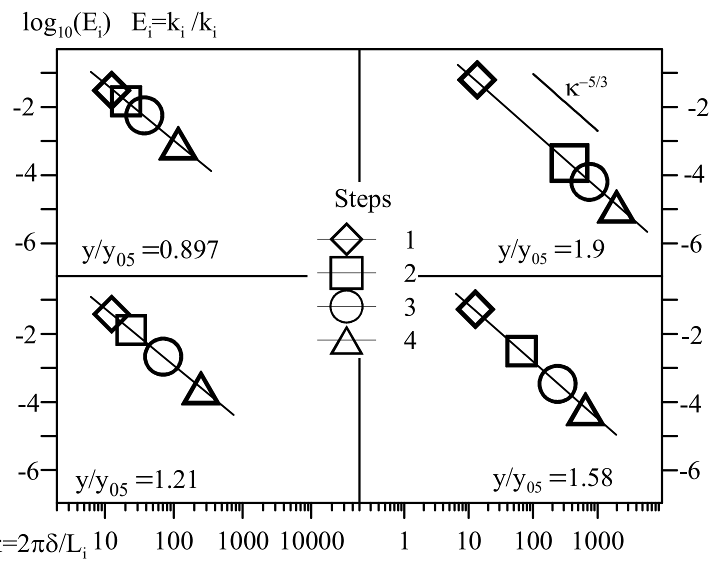

Fig.1. Calculation of 4 steps of cascade transfer in 4 cross serction of round jet .

The possibilities of calculating the near-wall flows were verified by simulations of the cascade process in the boundary layer. In this case, the flow in the boundary layer was calculated by the author's model [3], and the cascade transfere by the Jones-Launder model. The results are shown in Fig. 3.

The calculations of the correspondence of the spectrum to the "-5/3" law for several well-known models are shown in Figure 4.

Chen's model [6] shows that in the third step of the cascade process the wave numbers of the fluctuations decrease, i.e. the diameters of vortices increase. This is impossible. The Menter’s SST model [7] does not demonstrate any orderliness of the results by the wave numbers in the steps of the process. Launder-Sharma [8] model in the third step shows scales of turbulence on the order of $\left(10^{-8} \div 10^{-9}\right)\delta$. This is why the third and fourth steps are not shown on the plot. But such scales are much smaller than Kolmogorov's scale.

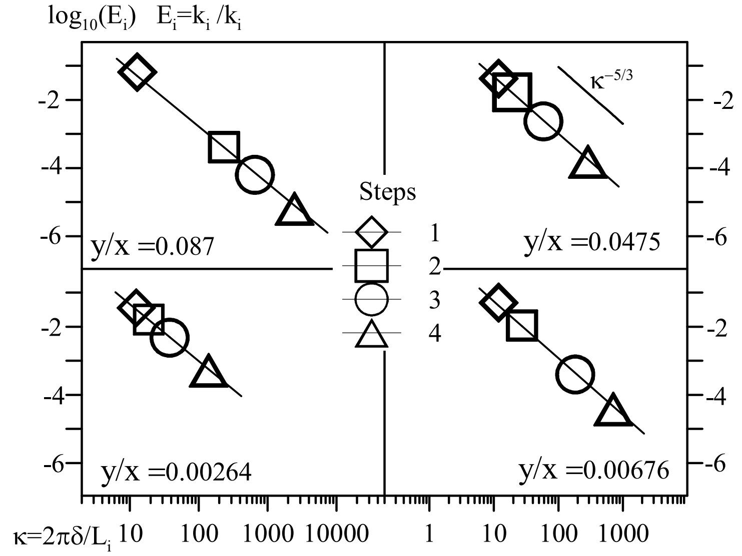

Fig.2. Calculation of 4 steps of cascade transfer in 4 cross serction of plane mixing layer .

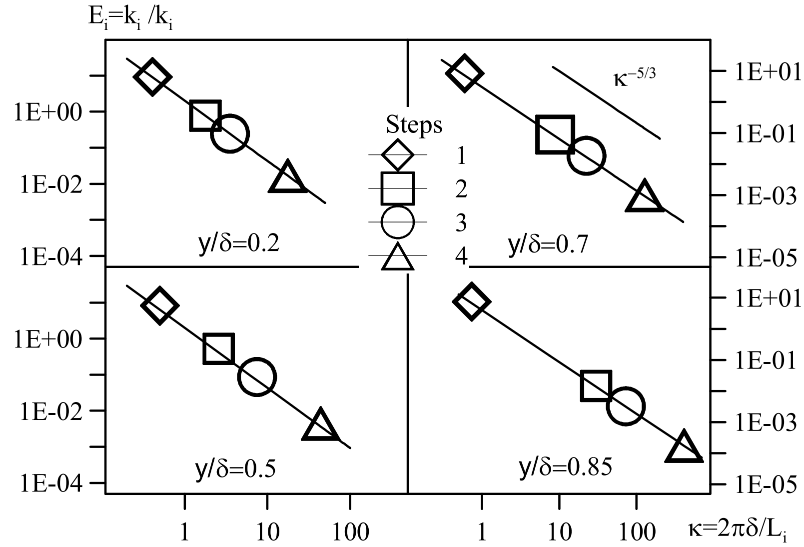

Fig.3. Calculation of 4 steps of cascade transfer in 4 cross serction of boundary layer.

Note that the Wilcox's model [8] correctly shows changes of the energy function in the cascade process steps.

Consider what this model has in common with the Jones-Launder model.

We can say that almost all k-ε models are the Jones-Launder model with corrections. A typical k-ε model can be written as

$$\begin{cases} \dfrac{Dk}{D\tau} = \dfrac{\partial}{\partial x_i}\left(\nu + \dfrac{\nu_t}{\sigma_k}\right)\dfrac{\partial k}{\partial x_i} + (P - \varepsilon) - E_k, \\ \dfrac{D\varepsilon}{D\tau} = \dfrac{\partial}{\partial x_i}\left(\nu + \dfrac{\nu_t}{\sigma_\varepsilon}\right)\dfrac{\partial \varepsilon}{\partial x_i} + \dfrac{\varepsilon}{k}\left(C_{\varepsilon 1} f_1 P - C_{\varepsilon 2} f_2 \varepsilon\right) + E_\varepsilon, \\ P = \tau_{ij}\left(\dfrac{\partial U_i}{\partial x_j}\right). \end{cases}$$

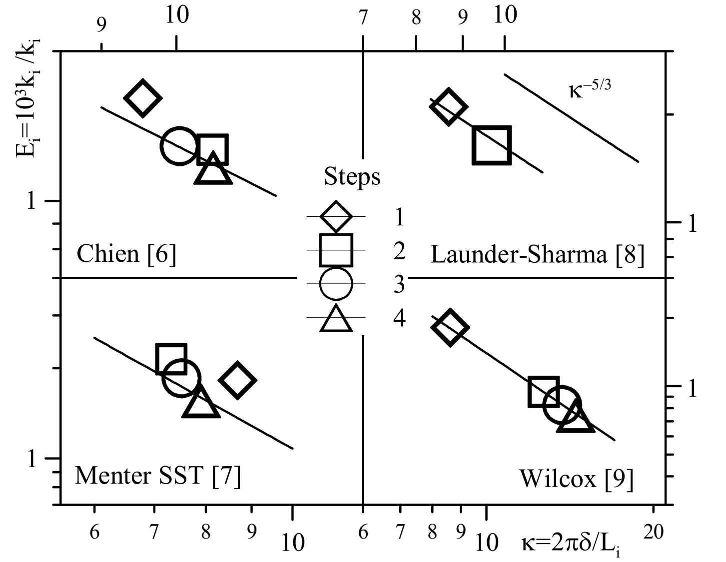

Fig.4. Calculation of 4 steps of cascade transfer in boundary layer by 4 models of turbulence .

Here $P = \tau_{ij}\left(\partial U_i/\partial x_j\right)$is the rate of energy transfer from the mean flow to turbulence or the production of turbulence energy.

The corrections E_k and E_ε in fact take into account the near-wall effect of viscosity on diffusion and are not considered further. In most models, the correction $f_1 = 1$. The model is mainly determined by the f_2 correction. Note that at a distance from the wall $f_1 = f_2 = 1$, $E_k = E_\varepsilon = 0$, that is, the model coincides with the Jones-Launder model.

Wilcox's model can also be classified as a Jones-Launder model, but with a different dissipative variable.

Only in the Jones-Launder and Wilcox models, the correction $f_2 = 1$ in the whole calculation area. In the author's opinion, it is this fact that allows these models to reproduce the "-5/3" law. Unfortunately, the author failed to prove this mathematically.

Why are these corrections needed?

The meaning of the corrections introducing is obvious. It is noted that in the near-wall region the Jones-Launder model gives strongly overestimated values of the turbulence energy. Therefore, in order to decrease the energy near the wall, it is necessary to decrease the term in the energy transfer equation in this region. But all the terms of this equation, except for the dissipation rate, are exact and their profiles are verified by experiments. Therefore, one cannot change them. Therefore, in all models, corrections f_1 and f_2 are introduced into the dissipation rate transfer equation, so that the dissipation rate at the wall increases. For this, f_2 at the wall should decrease (as a result, the dissipative term of the dissipation rate transfer equation in the wall region will decrease), and f_1 at the wall should increase, which will increase ε in this region. It is clear that these corrections in no way follow from the Navier-Stokes equations.

As a result, we have a contradiction that cannot be resolved within the framework of the traditional approach. To reproduce the "-5/3" pattern the corresponding terms of the model equations must have the form $(P-\varepsilon)$ and $(C_1P - C_2\varepsilon)$, but in order to reproduce the near-wall turbulence, it is necessary to introduce corrections into the dissipation transfer equation.

In [3], the author succeeded in resolving this contradiction. The proposed model has the form

$$\begin{cases} \dfrac{Dk_0}{Dt} = f_0 \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_{t0}}{C_k}\right)\dfrac{\partial k_0}{\partial x_k} + \left(f_0 P - \varepsilon_0\right), \\ \dfrac{D\varepsilon_0}{Dt} = f_0 \dfrac{\partial}{\partial x_k}\left(\nu + \dfrac{\nu_{t0}}{C_\varepsilon}\right)\dfrac{\partial \varepsilon_0}{\partial x_k} + \dfrac{\varepsilon_0}{k_0}\left(C_1 f_0 P - C_2 \varepsilon_0\right). \end{cases}$$

It has already been said that in order to decrease the energy at the wall, the term $(P-\varepsilon)$ must be reduced. In the traditional approach, this is done by increasing ε in the near-wall region. But at the same time, the capabilities of the model are noticeably reduced. At the same time, it is also possible to decrease $(P-\varepsilon)$ while not changing $(C_1 P - C_2 \varepsilon)$ by decreasing P, which was done in []. Here f_0 is a correction function equal to zero on the wall and 1 in free flow. This approach is in good agreement with a variety of experimental data. Many calculations carried out using this model are not available to modern models in principle.

6. Conclusions

The paper analyzes the reasons of difficulties in modeling the cascade process of turbulent energy transfer. According to the simulation results, no one modern model can correctly reproduce the cascade transfer. It is shown that the reasons that do not allow modeling the cascade transfer lie in the base of models designing method. It is argued that proposed by the author the new methodology for models designing effectively copes with these difficulties.

References

1. Jones W. P., Launder B. E.. The prediction of laminarization with a two-equation model of turbulence/ W. P. Jones and B. E. Launder// Int. J. Heat Mass Transfer. - 1972 - V. 15. - 301-314.
2. Speziale C., Bernard P.. The energy decay in self-preserving isotropic turbulence revisited/ C. Speziale, P. Bernard // J. Fluid Mech. - 1992 - V. 241. - 645-667.
3. Golovnya, B.P. Important properties of turbulent near-wall flows which are not accounted by modern RANS models / B.P. Golovnya // Int. J. Heat Mass Transfer. – 2020. – V. 146. – 118813.
4. Schiestel R. Multiple-time-scale modeling of turbulent flows in one-point closures / R. Schiestel R. // Phus. of Fluids. - 1987 - V. 30 - 722
5. McGuirk J. J..W. Rodi W. The Calculation of Three-Dimensional Turbulent Free Jets. / J. J. McGuirk, W. Rodi // Turbulent-Shear-Flows-1- Pennsylvania, USA-1977
6. Chien K.-Y. Prediction of channel and boundary-layer flows with a low Reynolds number turbulence model / K.-Y. Chien // AIAA J.-1982 - V. 20 - 34–38.
7. Menter F.R. Zonal two-equation k-ω turbulence model for aerodynamic flows./ F.R. Menter // AIAA Paper. - 1993 - V. 93 - 2906
8. Launder B.E., Sharma B.I. Application of the energy-dissipation model of turbulence to the calculation of flow near a spinning disc. / B.E. Launder, B.I. Sharma, // Lett. Heat Mass Transfer - 1974 - V.1 - 131–138.
9. D.C. Wilcox, Turbulence Modelling for CFD. / D.C. Wilcox // DCW Industries, Clfrn. USA - 1994.

ГОЛОВНЯ Борис Петрович,

ДО ПИТАННЯ ПРО КАСКАДНЕ ПЕРЕНЕСЕННЯ ЕНЕРГІЇ В ТУРБУЛЕНТНИХ ТЕЧІЯХ

Каскадне перенесення енергії відіграє найважливішу роль у всіх турбулентних течіях. На жаль цей процес дуже погано піддається моделюванню. У даній роботі аналізується одна з можливих причин виникнення труднощів. Показується, що модель турбулентності, запропонована раніше автором, з цими труднощами успішно справляється

Ключові слова: моделювання каскадного перенесення, $k-\varepsilon$ модель турбулентності.

Одержано редакцією 11.07.2018
Прийнято до публікації 20.09.2018