СЕКЦІЯ «ПРИКЛАДНА МАТЕМАТИКА»

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SOME SYSTEMATIC MISTAKES IN NEAR-WALL TURBULENCE MODELING AND POSSIBLE WAY TO OVERCOME THEM

All near-wall models of turbulence do not reproduce the cascade energy transfer. In the author opinion, models poorly take into account the structure of near-wall turbulence. Therefore *these models can not reproduce correctly all processes in turbulent boundary layer. Traditional correction terms can not be regarded as physically reasonable solution of the problem. As a result there are many turbulent flows that can not be simulated. This paper presents a possible explanation of difficulties encountered in the development of turbulence models for the calculation of the boundary layer. A technique to overcome these difficulties is presented.*

Keywords: *turbulent boundary layer, near-wall turbulence modeling.*

Introduction

Almost all modern models of turbulence are based on the equation of turbulent energy transfer. If such model is reckoned as universal then it must answers the three questions. First – where does the turbulent energy originate? Second – what happens to this energy later? Third – how does the energy dissipate? Otherwise there are certainly exists such flow that this model can not reproduce.

Kolmogorov's theory of cascade energy transfer answers these questions. By this reason results of simulations by high-quality models must be in agreement with this theory.

It is known that the RANS and LES type models do not reproduce the basic features of cascade energy transfer in simulations of the boundary layer. Main reason – these models poorly take into account the structure of near-wall turbulence.

Physical effects not taken into account in modern models

Experimental investigations [1], [2], [3] show that main role in a turbulent boundary layer is played by long-scaled quasi-ordered vortex structures commensurable with the layer thickness. Structures are disposed across the flow and have a limited length. The longitudinal size of such a structure is $\Delta x \approx 1.6\delta$ [2] or $\Delta x^{+} \approx 1000$ [4], [5]. The distance along the stream in which it retains its own characteristics is about of 5δ [6], Convection velocity of these structures is $U/U_e \approx (0.46 - 0.62)$ [7], [8], and U increases with increasing of Re [8].

Their schematic representation is shown in Figure 1 [3]. Structures move along the flow. While the structure moves velocity at a fixed point of the space within this structure gradually decreases in time. After passing the boundary that closes this structure, intense high-frequency fluctuations of velocity, temperature, pressure, etc. are observed and there occur jet ejections of decelerated liquid from the wall and invasion of accelerated to the wall region [9]. Experiments show that these jet ejections of liquid are accompanied by the long-scaled vortexes deceleration.

As a result of the interaction of accelerated and decelerated parts of two long-scaled structures an inflection point appears on the average velocity profile. In turn, the appearance of the inflection point provides the appearance of new transversely oriented vortex [10]. Thus the transverse vortex originates and therefore exists at the expense of the energy of the averaged flow. All this is one cycle of a turbulent boundary layer renewal.

The ejections interact with the averaged flow. As a result very thin longitudinally oriented vortexes are formed from the ejections. These vortexes merge pairwise and form so called horseshoe vortexes [11]. Horseshoe vortices intertwined with each other. Their location according to [12] is shown in Figure 2.

Experiments show that time scales of theses longitudinally oriented and horseshoe vortexes are rather small. But in the development of turbulence models a very large time of averaging is used. Theoretically this time tends to infinity. As a result of such averaging the following picture arises from the described process.

This intertwining of horseshoe vortexes is observed as comprehensive whole, not as collection of separate eddies. We will call it a medium-scale structure. However, this structure really consists of thin vortexes. So, main properties of such vortexes are applied to the structure. First. In accordance with Kolmogorov's theory, thin vortexes dissipate very rapidly. So, the rate of dissipation in the medium-scale structure must be very high. Second. Thin vortexes create very small mixing. So, the turbulent viscosity in medium-scale structure is very small.

At the same time simulations show that medium-scale structures have noticeable turbulent energy. For example, the model for calculation of the development of disturbances in the boundary layer was proposed by Zhang and Lilley [13]. In this model periodic disturbances were imposed on a turbulent flow. As a result, secondary disturbances in the form of slowly rotating formations appeared in the flow. These formations are very similar to the aggregation of horseshoe vortexes. They are placed in region $y_+ \leq 100$ and create fluctuations of averaged velocity $\Delta U_+ \approx 0.7 \div 0.75$. So, increase of the turbulent energy can be estimated as $\Delta k_+ \approx 0.25 \div 0.28$.

Now consider the turbulent energy spectrum in the boundary layer. It is supposed that a) the spectral function is averaged by time; b) long-scaled structures obey Kolmogorov's law

It is known, that in free stream the *k*-*ε* type model satisfactorily reproduces inertial part of turbulence spectrum, i.e. results of calculations are in accord with Kolmogorov's '-5/3' law (for example, see [14]). The schematic plot of the spectral function is shown in Figure 3a.

The hypothetical graph of the spectral function in some cross section of the layer which is placed very close to the wall is shown in Figure 3b. Figure 3a and 3b differ very markedly. Form of the Figure 3b is determined by two factors. First, main part of the graph corresponds to the cascade process theory because main part of the turbulent energy is concentrated in the long-scale structures. This part of the graph is in agreement with Figure 3a. Second, the ejection is presented in the graph at high wave numbers. It is determined by existence of the medium-scale structures in the layer.

Let's move the cross section from the wall. Sizes of the medium-scale structures are much smaller than the layer thickness. So, at some distance from the wall the effect of these structures must start to decrease with increase of the distance, i.e. size of the ejection in Figure 3b must decrease also. On the boundary of layer Figure 3a and 3b must coincide.

Any sufficiently general model of turbulence must be agreed with such behavior of turbulence. This means that model must answer the following questions.

1. From the point of view of the theory of cascade process Figure 3b violates the energy conservation law. Indeed, the cascade process guarantees that if the wave number increase then the turbulent energy decreases. Growth of the turbulent energy in the cascade process is impossible
2. As it was said above, the experiments show that the jet ejections of the liquid are accompanied by deceleration of the long-scaled structures. Because the medium-scale structures appear from these ejections it is natural to assume that some part of the braking energy is used on production of the medium-scale structures. So, it can be said that direct energy transfer from the long-scale to the medium-scale structures exists in the layer and this transfer occurs without any involvement of the cascade process. It is not clear how to introduce this mechanism into the model, especially if this model is in agreement with the cascade process.
3. In RANS models fluctuation flow is averaged by time. In LES modes average is replaced by filtering. But as it is known filtering in LES method can be regarded as spatial average (for

example, see [15]). In any case very strange formation appears on location of the medium-scale structure. It has sufficiently large sizes $(y_+ \approx 100)$, noticeable turbulent energy, dissipate very rapidly, and create vanishingly small turbulent viscosity.

All authors of models of turbulence in fact attempt to correspond to third item of this list. That means that all models of turbulence have the following properties.

1. The calculated turbulent energy is equal to the sum of the energies of the long-scale and medium-scale structures.
2. The calculated dissipation rate in the near-wall region is sufficiently high. This simulates the rapid dissipation of the medium-scale structures.
3. The calculated turbulent viscosity in the near-wall region is strongly suppressed. This simulates the fact that the medium-scale structures have not effect on the turbulent viscosity.

These properties are obtained by introducing into the model of the complex correction functions. Unfortunately these corrections effect on the long-scaled structures also. As a result the model looses all original positive qualities of the models without corrections and can not simulate the cascade process.

New approach to turbulence modeling

The author offers a radically different approach. It is based on the following hypothesis. From an engineering point of view the main problem of the turbulence modeling is to provide correct simulation of the mean flow. In its turn, quality reproduction of the turbulent viscosity guarantees correct simulation of the mean flow. But as stated above, the medium-scale structures create vanishingly small turbulent viscosity. So their influence can be neglected. At the same time sizes of the long-scale and medium-scale structures differ very markedly. By this reason the medium-scale structures weakly effect on the long-scale structures and this effect can be neglected also. So we can eliminate the medium-scale structures from the model without loss of generality.

The medium-scale structures can be eliminated in the following way. Let k be the total turbulent energy, k_0 – the energy of the long-scale structures, and k_1 – the energy of the medium-scale structures. Neglecting the energy of interaction of structures we get $k = k_0 + k_1$. Denote $f_0 = k_0/k$ or $k_0 = f_0 k$. Here f_0 – some function.

Schematically the equation of total turbulent energy transfer can be written as

$$Dk/Dt = Diff(k) + P - \varepsilon \tag{1}$$

Here *Diff* – operator of diffusion transfer, P – rate of turbulent energy production, ε - rate of k dissipation. Multiplying (1) on function f_0 we obtain

$$\left\{ f_0 \frac{Dk}{Dt} = f_0 Diff(k) + f_0 P - f_0 \varepsilon \right\} = \left\{ \frac{Df_0 k}{Dt} = Diff(f_0 k) + f_0 P - f_0 \varepsilon \right\} = \\ = \left\{ \frac{Dk_0}{Dt} = Diff(k_0) + f_0 P - \varepsilon_0 \right\} \tag{2}$$

Here $\varepsilon_0 = f_0 \varepsilon$ - dissipation rate in the long-scale structures. Major result – the production term is multiplied on additional function. Transformation $f_0 Diff(k) = Diff(f_0 k) = Diff(k_0)$ is the model of turbulent diffusion.

Let us remark that all these calculations are not rigorous and are needed only to clarify the origin of the equation (2). The same result can be obtained strictly on the base of equations of fluctuation components transfer.

Equations of the k-ε model for free flow are well-known

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_k}\right)\frac{\partial k}{\partial x_k} + P - \varepsilon, \tag{3}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_k} + \frac{\varepsilon}{k}\left(C_1 P - C_2\varepsilon\right), \tag{4}$$

$$\nu_t = C_\nu \frac{k^2}{\varepsilon} \tag{5}$$

$$P \equiv -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \tag{6}$$

.

Now, taking into account equation (2), let us introduce corrections into the system (3)-(5). The resulting system has the following form

$$\frac{Dk_0}{Dt} = \frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_k}\right)\frac{\partial k_0}{\partial x_k} + f_0 P - \varepsilon_0 - E_k, \tag{7}$$

$$\frac{D\varepsilon_0}{Dt} = \frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_\varepsilon}\right)\frac{\partial \varepsilon_0}{\partial x_k} + \frac{\varepsilon_0}{k_0}\left(C_1 f_0 P - C_2\varepsilon_0\right) - E_\varepsilon \tag{8}$$

$$\nu_t = C_\nu F_\nu \frac{k_0^2}{\varepsilon_0} \tag{9}$$

Here E_k and E_ε are the corrections assigned to balance the diffusion on the wall.

To close model (7)-(9) it must first be supplemented by the expression for the function f_0. The graph of this function can be obtained based in the experimental data. In doing so it is enough to substitute into the model the experimentally obtained distributions of the turbulence energy and eddy viscosity and to consider system (7)-(8) as the system of equations relative to the function f_0 and dissipation. It is supposed hear that $k_0 = k$ and $E_k = E_\varepsilon = 0$. Because we just want to understand how this graph looks this assumption does not exercise a significant influence.

But it is much more convenient to generate the values of the turbulence energy and eddy viscosity by calculating on the basis of the well-tested models of turbulence. In the present paper, the models of Chien [16], Launder and Sharma [17], and Nagano and Tagawa [18] are used as data generators. The results of calculations of the function f_0 are given in Figure 4. It is of interest to note that the graph almost exactly coincides with the velocity distribution in a laminar boundary layer. For comparison Figure 4 gives the familiar Pohlhausen solution.

As a result of variance calculations the following approximation was obtained for the function f_0:

$$f_0 = \left(1 - \exp\left(-\mathrm{Re}_{y0}/5.5\right)\right)\left(1 - \exp\left(-2.4\, y/L_{\varepsilon 0}\right)\right) \tag{10}$$

where $Re_{y0} = \sqrt{k_0}\, y/\nu$, $L_{\varepsilon 0} = k_0^{3/2}/\varepsilon_0$. Test calculations have shown that the expressions

$$E_k = (1 - f_0)\frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_k}\right)\frac{\partial k_0}{\partial x_k}$$

$$E_\varepsilon = (1 - f_0)\frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_\varepsilon}\right)\frac{\partial \varepsilon_0}{\partial x_k}$$

are good approximation for the wall corrections.

After substitution of the wall corrections into the system of equations and cancelation we obtain the form of presentation of model equations

$$\frac{Dk_0}{Dt} = f_0\frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_k}\right)\frac{\partial k_0}{\partial x_k} + f_0 P - \varepsilon_0, \quad (11)$$

$$\frac{D\varepsilon_0}{Dt} = f_0\frac{\partial}{\partial x_k}\left(\nu + \frac{\nu_t}{C_\varepsilon}\right)\frac{\partial \varepsilon_0}{\partial x_k} + \frac{\varepsilon_0}{k_0}\left(C_1 f_0 P - C_2\varepsilon_0\right) \quad (12)$$

By the results of test calculations for the function F_ν we obtained the following approximation:

$$F_\nu = \left(1 - \exp\left(-\mathrm{Re}_{y0}/45\right)\right)\left(1 - \exp\left(-2.4\, y/L_{\varepsilon 0}\right)\right) \quad (13)$$

The constants and the boundary conditions are:

C_ν=0.09, C_ε=1.3, C_k=1, C_2=1.45, C_1=0.9C_2.

$y=0$ - $k_0=\varepsilon_0=0$, $y\to\infty$ - $k_0=k_e$, $\varepsilon_0=\varepsilon_e$

For constants C_ν, C_ε, and C_k the standard values were used. The constant C_2 has been chosen in the following way. Transfer of turbulence in the external flow is described by the system of equations with initial conditions

$$U_e\frac{dk_e}{dx} = -\varepsilon_e,$$

$$k_e(x_0) = k_0,\ \varepsilon_e(x_0) = \varepsilon_0.$$

$$U_e\frac{d\varepsilon_e}{dx} = -C_2\frac{\varepsilon_e^2}{k_e}$$

This system has an exact solution

$$k_e = k_0\left(\frac{\tilde{x}}{x_a}\right)^{\frac{1}{1-C_2}},\ \varepsilon_e = \varepsilon_0\left(\frac{\tilde{x}}{x_a}\right)^{\frac{C_2}{1-C_2}},$$

where

$$x_a = \frac{k_0 U_e}{\varepsilon_0 (C_2 - 1)}, \quad \tilde{x} = x_a - x_0 + x .$$

The flow in the free stream becomes laminar at infinity. By this reason at infinity turbulent energy, rate of dissipation and dissipative scale are equal to zero. So

$$\lim_{x\to\infty} \frac{k_e^{3/2}}{\varepsilon_e} = \lim_{x\to\infty} \frac{k_0^{3/2}}{\varepsilon_0} \left(\frac{\tilde{x}}{x_a} \right)^{\frac{1.5-C_2}{1-C_2}} = 0$$

As a result we have $(1.5 - C_2)/(1 - C_2) < 0$ or $1 < C_2 < 1.5$. Constant C_2=1.45 was taken on the base of this relation.

Testing the *k–ε* model: results of the calculation of forced turbulent flow in a boundary layer

Figure 5, 6 and 7 show the results of calculations of a zero pressure gradient flat plate turbulent boundary layer. The agreement with the experimental data is very good. It will be recalled that the medium-scale structures were ignored in these calculations. By this reason the calculated values of the turbulent energy are less then the experimental data.

Cascade energy transfer

Calculation of cascade energy transfer may be performed in the following way. At the first step, the system of the *k–ε* type is solved. At the second step, the same system is solved with the dissipation, which is obtained at the first step, being used instead of the generation term. It is evident that the calculation step simulates the step of the cascade process. The number of steps is not limited.

The correspondence of the results of calculation to the cascade process can be verified in the following way. First, it is known that transition of turbulent energy to heat occurs in vortices that have the dimensions commensurable with the Kolmogorov scales, i.e., the energy transferred to the cascade does not change. Hence it follows that if the dissipative term reflects the transfer of energy into the cascade process, then in such calculation it must not change.

Second, if we assume that the wave number is proportional to $1/L_\varepsilon$, which follows from the dimensional reasons, then the dependence of k_i/k on $1/L_i$, where i is the number of the step of the mentioned calculation, must not, as a minimum, be in contradiction with the known regularities of the cascade transfer.

In the calculations by this model, the requirement ε=const is satisfied almost exactly (see Figure 8). At the same time, the turbulence energy, dissipative scales, and scales of time decrease noticeably from step to step.

The dependence of k_i/k on $1/L_i$ is shown in Figure 9. It is in good agreement with the "-5/3" law.

In the judgment of the author, these results prove that in the present model *ε* reproduces just the rate of energy transfer to the cascade process. In other words, *ε* has a specific physical meaning, which cannot be said of the traditional models. In particular calculation of the cascade process by the Nagano–Tagawa model [18] showed that in this case, in development of the cascade process the vortices are shifted toward higher, rather that lower, wave-numbers.

Calculation of the boundary-layer flow with a large positive pressure gradient

Model (11)-(12) was verified by calculations of the boundary-layer flow with a large positive pressure gradient (experiments of Samuel and Joubert [19]). Figures 10 and 11 present the results of calculations of the friction coefficient and the profiles $\overline{uv}$ in three cross sections of the channel. The correlation $\overline{uv}$ was calculated according to the Boussinesq hypothesis. The agreement with the experimental data is satisfactory (X streamwise coordinate, m).

References:

1. Brown, G.L. and Thomas, S.W. (1977) Large structure in a turbulent boundary layer. Phys. Fluids. V. 20, № 10, 243-252.
2. Falco R.E. (1977) Coherent motions in the outer region of turbulent boundary layers. Phys. Fluids. V. 20, № 10, 124-132.
3. Nakagawa H., Nezu I. and Matsumoto N. (1980) Structure of space-time correlations of bursting phenomena in an open-channel flow. Mem. Fac. Eng. Kyoto Univ. V. 42, № 1. 85-124.
4. Blackwelder R.F., and Eckelmann H. (1979) Streamwise vortices associated with the bursting phenomenon. J. Fluid. Mech. V. 94, 577-594.
5. Eckelmann H. and Nychas S.G. (1977) Vorticity and turbulence production in pattern recognized turbulent flow structures. Phys. Fluids. V. 20, № 10. 225-231.
6. Roshko A. (1976) Structure of turbulent shear flows: a new look. AIAA. J. V. 14, № 10, 1349-1357.
7. Kreplin H.-P. and Eckelmann H. (1979) Propagation of perturbations in the viscous sublayer and adjacent wall region. J. Fluid. Mech. V. 95. Pt. 2.
8. Rajagopalan S. and Antonia R.A. (1979) Some properties of the large structure in a fully developed turbulent duct flow. Phys. Fluids. V. 22, № 4, 614-622.
9. Blackwelder R.F.and Kaplan R.F. (1976) On the wall structure of the turbulent boundary layer . J. Fluid. Mech. V. 76., Pt. 1. 89-112.
10. Offen G.R and, Kline S.J. (1975) A proposed model of the bursting process in turbulent boundary layers // J. Fluid. Mech. V. 70, Pt. 2, 209-228.
11. Head M.R. and Bandyopadhyay P. (1981) New aspects of turbulent boundary-layer structure. J. Fluid. Mech. V. 107, 297-338.
12. Utami T. and Ueno T. (1987) Experimental study on the coherent structure of turbulent open-channel flow using visualization and picture processing. J. Fluid. Mech. V. 174, 399-440.
13. Zhang Z. and Lilley G.M. (1982) A Theoretical Model of the Coherent Structure of the Turbulent Boundary Layer in Zero Pressure Gradient. In: Durst.F. et al.,Ed., Turbulent Shear Flows 3, Springer-Verlag, P.60-72.
14. Speziale C. and Bernard P. (1992) The energy decay in self-preserving isotropic turbulence revisited. J. Fluid Mech. V. 241, 6445-667.
15. Berselli L., Iliescu T.and Layton W. (2006) Mathematics of Large Eddy Simulation of Turbulent Flows. Springer-Verlag Berlin Heidelberg.
16. Chien K.-Y. (1982) Prediction of Channel and Boundary-Layer Flows with a Low-Reynolds-Number Turbulence Model. AIAA J., V.20, 34-38.
17. Launder B.E.,and Sharma B.I. (1974) Application of the Energy-Dissipation Model of Turbulence to the Calculation of Flow Near a Spinning Disc. Let. in Heat and Mass Transfer, V. 1, 131-138.
18. Nagano Y. and Tagawa M. (1990) An improved (k-ε) Model for Boundary Layer Flows. J. of Fluid Eng. V. 112, 33-39.
19. Samuel, A.E. and Joubert, P.N. (1974) A Boundary layer Developing in an Increasingly Adverse Pressure Gradient. J.Fluid Mech. V. 66, №3, 481-506.

Анотація

Б.П. Головня

Деякі систематичні помилки в моделюванні пристенной турбулентності і можливи шляхі їх подолання

Всі моделі пристенной турбулентності жодного разу не відтворюють каскадний перенос енергії турбулентності. На думку автора, моделі погано враховують структуру пристенной турбулентності. Тому ці моделі не можуть відтворити правильно все процеси в турбулентному прикордонному шарі. Традиційні поправочні члени не можуть розглядатися як фізично розумне рішення цієї проблеми. В результаті існує багато турбулентних течій, які не піддаються модельованню. Ця стаття являє собою можливе пояснення труднощів, що виникають при розробці моделей турбулентності для розрахунку прикордонного шару.

Ключові слова: *турбулентний прикордонний шар, моделювання пристеночной турбулентності.*

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